Mathematics Lessons for Grades 9-12

“Why Doesn't SSA (side, side, angle) Work?”

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Discipline: Geometry
Grade: 9 to 10

NCTM Standards
Geometry (Understanding why SSA is not a valid congruence relation for triangles)
Specific Geometry Standards:
Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships
Use visualization, spatial reasoning, and geometric modeling to solve problems
Reasoning & Proof (Students will be making two different constructions to show that SSA is not always a valid congruence relation for triangles.)
make and investigate mathematical conjectures;
develop and evaluate mathematical arguments and proofs;
Representation: create and use representations to organize, record, and communicate mathematical ideas;

Purpose/Goal
Students will be able to explain when SSA does work, when SSA does not work, and give a proof by contradiction that SSA does not always work.

Context
Geometry students are often forgetful about which congruence relations are valid in Euclidean geometry. SSA, SAS and ASA (and HL) are the congruence relations taught in high school geometry classes. Students often forget and think that SSA is among that list. This lesson prompts students to prove that SSA does not work in all cases. It also goes further and shows students that sometimes SSA does work, and that HL is a special case of SSA.

Preparation
Students should have extensive knowledge on triangles, and know how to use a compass/straight edge to do basic constructions. Students should be nearing the end of the unit on triangle congruences. It would be helpful for students to understand the techniques involved in proof by contradiction, but this is not necessary.
This lesson fits with a strand of lessons on congruent triangles.

The teacher needs to have plenty of construction paper, markers, compasses, rulers, and several pipe cleaners (optional). There is an accompanying worksheet which students will fill out. This worksheet walks the students through a proof by contradiction that SSA works in all cases. It also clearly explains when SSA works (when the middle "S" is shorter than the first "S").

It would be a good idea, since the activity is somewhat involved, that the teacher complete the activity so that any questions will be easily answered during the lesson. I would also suggest that the teacher give some small examples of proof by contradiction the class previous to the activity.

Websites
The entire lesson can be found at:
http://www.ms.uky.edu/algebracubed/lessons/why_doesnt_ssa_work.pdf
The activity was inspired by:
http://mathforum.org/library/drmath/view/54659.html

Motivation
I would suggest that the teacher discuss proof by contradiction with a few basic examples. Suppose that I have eleven fingers. Look down and count your fingers. I have observed that I have ten fingers. Ten does not equal eleven. Thus I must have assumed wrong. I do not have eleven fingers.

Description
The lesson begins with reminding students of the correct congruence relations. If desired, a couple of examples of proof by contradiction may be given. The activity is self explanatory. After dividing the students into groups (or letting the students divide themselves), students should come and gather their materials and begin working. The teacher should circulate to make sure that students stay on task and don't have any questions. Some steps are trickier than others. (My students needed help on the first part with step 5.) It will be very important that the teacher go to each group and make sure that they follow the reasoning, and can identify the triangles which are "supposed" to be congruent by SSA. I found that it was much more effective to do this group by group than explaining it to the entire class because the students seemed to work at different paces. I made each student responsible for pointing out the supposedly congruent triangles, and explaining why they were not actually congruent. (ie: Explaining the "contradiction")

During part 2 students create another example of when SSA does work. This construction will go much more quickly, and was obvious to my students. During the wrap up, the most important questions were to ask students:
1. When does SSA work?
2. What is another special case where SSA works? (Hint: Let the A be a 90 degree angle.)
3. What was the contradiction when the middle "S" was longer than the first "S"? (In part 1).
4. What can you say in general about SSA as a congruence relation?

Assessment
The main mode of student assessment is via conversation with each individual group. I implemented this in two classes with around 32 students each. I had enough time to address each pair of students in a fifty minute class. Each pair completed the activity, and we had about two minutes for the wrap up questions at the end.

It may be interesting to put a question on a future assignment asking if two triangles are congruent by SSA. I would give bonus points to the student who remembered that when the middle "S" was shorter, SSA does work. However, it would also be correct if a student put that they were taught not to use SSA as a valid congruence relation.

Follow-Up Activities
To further explore this teachers may wish to implement more proofs by contradiction in their classroom. Teachers could also implement more constructions/visualizations into the geometry classroom.