Mathematics Lessons for Grades 9-12

"Topolo-what?"

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Discipline: Geometry (pre-viewing topology topics with high school geometry students to promote interest in three-dimensional figures)
Grade: 9 to 10

Standards:
NCTM Standards: Geometry (This is especially centered around 3-D Geometry)
This lesson meets the following Geometry standards:
Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.
Apply transformations and use symmetry to analyze mathematical situations
Use visualization, spatial reasoning, and geometric modeling to solve problems

Connections (Connections to Algebra/higher level mathematics is made throughout the lesson.)
Representation (Students will be making models (2-D and 3-D) to interpret mathematical phenomena.)

Purpose/Goal
Students should be able to understand the difference between homeomorphisms and quotient maps (on a VERY basic level), make Mobius strips and explain several transformations of different three-dimensional figures. Students should also be able to give several examples of sets. Students will also be introduced on a basic level to equivalence classes, and will demonstrate their knowledge by completing an activity sheet.

Context
Students often think that upper level college math is frightening, difficult to understand and overwhelming. The purpose of this lesson is to introduce an upper level mathematics course in a friendly environment and allow students to experiment with math they may never see unless they major in mathematics. Hopefully some students will find it interesting enough to choose a career in the mathematics field.

A basic understanding of three-dimensional figures is a pre-requisite for this lesson (definitions and illustrations). The theorem on the relationship between faces, vertices and edges is a nice concept to teach directly before this lesson because there is a discussion on Euler's contributions to mathematics at the beginning of the lesson, and this is one of them. Students will be working mostly on physically manipulating the three dimensional figures, so an in-depth knowledge is not required.

This lesson fits well at the beginning or the end of a unit on three-dimensional geometry. I taught this lesson at the beginning to promote student interest in the rest of the unit.

Preparation
Access to Powerpoint and a way of displaying it to the students.
Scissors
Several sheets of paper for students to create shapes with.
Activity Sheet prepared specifically for this lesson.
Cylinder manipulative which can be transformed to a cone by pulling the top circle to one point.
A Klein bottle would be a nice object to bring in, but they are often expensive and difficult to obtain.
(Any other interesting topology related materials!)

Website
There are some interesting Mobius strip activities at the following:
http://mathssquad.questacon.edu.au/mobius_strip.html

Motivation
The activity I would suggest is having students create a Mobius strip, and then ask them to cut it in half to make two Mobius strips. When the students cut the Mobius strip in half, it will only become one very large Mobius strip with even more twists in it. They would not have expected this to happen. This is a good way to start students thinking about this new, abstract field of mathematics.

The way I began my students thinking about this lesson is to think about their favorite mathematical puzzle and share it with the class. This is because the introduction I give to topology is that it was motivated by a famous puzzle.

Description
The lesson begins with a short discussion of mathematical puzzles. It moves to a discussion of what I do in graduate school at the University of Kentucky. I give a brief history (via Powerpoint) of Topology to get students motivated. I explain that Topology is centered around the idea of connections. I include in the Powerpoint very basic definitions of Topology, Homeomorphism, Quotient Maps, Equivalence Classes. Students will write these definitions down so that they can refer to them for the activity. After the Powerpoint I give illustrations of shapes that are homeomorphic to each other (mostly two-dimensional) by drawing them. I will discuss quotient maps by providing the example of a cylinder having it's top circular base mapped to one point resulting in a cone. I will then have students create circular strips and Mobius strips (cutting the circular strip to get two, cutting the Mobius strip to get one longer, more twisted Mobius strip). I will suggest that students who are interested try to make more shapes by changing the way they forme their Mobius strip, the way they cut it, and perhaps to try putting two together to start with. (We do not have time to do all of this in class). I will then discuss that by gluing two Mobius strips together on all of their edges, a Klein bottle is created. I will provide the limerick:

A mathematician named Klein
Thought the Möbius band was divine.
Said he: "If you glue
The edges of two,
You'll get a weird bottle like mine."

I show an image of a Klein bottle, but do not have a physical Klein bottle.

Students will then complete an activity sheet asking them to classify the English alphabet into homeomorphism equivalence classes. I wrap the class up by asking students to explain the difference between homeomorphisms and quotient maps (no gluing allowed, gluing allowed). I will ask students to explain two to three things they learned during the lesson. If there is time at the end, I will have students try to construct a Klein bottle by taking a square and identifying the appropriate sides with the appropriate orientation.

Assessment
The assessment in this activity is the discussion at the end as well as the activity sheet/completion of the Mobius bands. The big question to answer is: What is the difference between a two homeomorphic spaces and two spaces related by a quotient map?

Follow-Up Activities
This could easily be explored further by going into more details in any of the definitions/concepts. However, the point of the lesson is to provide a general overview to get students interested in higher level mathematics. It is important that students have a good understanding of basic mathematics at the high school level. So, this lesson should be motivation for students to understand high school curriculum. If students need further motivation, perhaps an interactive/introductory lesson on Modern Algebra/Coding Theory/Analysis would be intersting to try!